

Femtosecond Laser Micro Machining

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Abstract

The work done in this paper is using the mathematics model we created to simulate the micro-machining process of femtosecond laser. made the following results:

- Searched and built mathematics differential equation that can be used in the program
- Drew a 3D graph about the relationship with laser's radius and the density of free electrons, which is the standard we used to check whether the micro-machining is finished or not using Matlab
- Wrote C++ program to simulate the process in a shorter running time than Matlab code

1 Introduction

The reason why we choose femtosecond laser as our tool to do the micro-machining is because of its property. Femtosecond lasers are ultra-short pulsed lasers with pulse durations in the femtosecond range and focused peak powers of up to even $10^{21} \frac{W}{cm^2}$.

Figure 1¹ illustrates the time-domain evolution of the femtosecond laser interaction with a semiconductor material. During the interaction between femtosecond lasers and semiconductor materials, the photon energy is absorbed by the material in the hundred femtosecond scale and excites carriers inside the material; subsequently, in the picosecond scale, the energy absorbed by the material is transferred to the lattice through carrier-phonon coupling; After the time scale reaches the nanosecond level, the thermal equilibrium between the carriers and the lattice is reached and the material starts to produce phase changes (ablation, evaporation, reconsolidation, etc.). The above process is much longer than the pulse duration of a femtosecond laser, which means that during the duration of the femtosecond pulse, the energy transfer between the electrons and the lattice is not yet complete, and the material does not fully complete the phase transition process.

¹Sundaram S K, Mazur E, Inducing and probing non-thermal transitions in semiconductors using femtosecond laser pulses[J]. Nature Materials, 2002, 1(4): 217

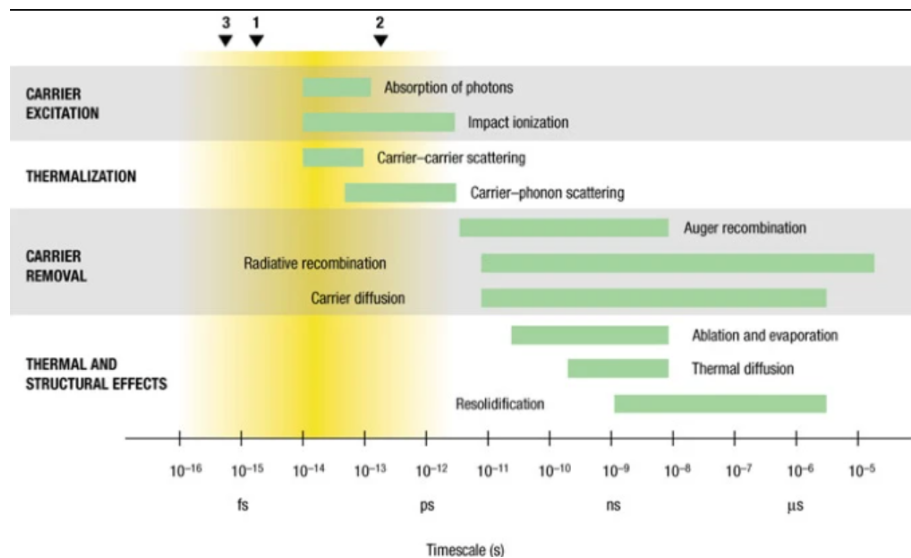


Figure 1: TimeScale

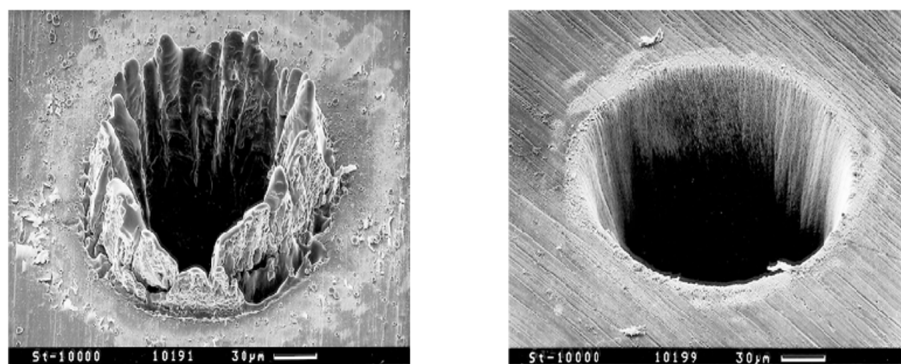


Figure 2: Machining holes using 3.3ns(left) and 200fs(right)

Chichkov et al. compared picosecond lasers with femtosecond. The difference in the shape of the laser hole in the stainless steel material is shown in Figure 2². It can be seen that the femtosecond laser processing is. The micropore inlet surface obtained is smoother, cleaner, and the heat affected zone is smaller. The use of femtosecond laser processing can effectively suppress. The thermal diffusion of the material reduces the generation of debris and recast layers in the subsequent phase transformation process.

2 Methodology

2.1 Principle of Building Mathematical Model

During the action of the femtosecond laser pulse and the material, the electrons in the material gain excited energy by way of multiphoton nonlinear absorption of the incident laser. The energy gained is rapidly accumulated in the absorption layer of only a few nanometers in thickness, and the temperature in the action area is instantaneously and dramatically increased and far exceeds the melting and vaporization temperature threshold of the material, causing the material to be highly ionized and eventually in an unparalleled high temperature, high pressure and high density plasma state.

Based on this principle, we can know that the material will be ionized by the laser and will be discharged as a form of free electrons. In that case, if the density of free electrons reach some points, we can conclude that the material is finished micromachining.

2.2 General Formula

When we are trying to build a differential equation about the density of free electrons, we need to understand that free electrons are consists of 2 classes: ³

- The electron absorbs enough photons to achieve ionization – class 1
- Free electrons from class 1 hit other electrons, ionizing other electrons experimentally – class 2

Based on these 2 classes, we can come up with a general partial differential equation between the density of free electrons and laser's lasting time:

$$\frac{\partial n_e(t, r, z)}{\partial t} = \beta(I)n_e(t, r, z) + P(I)$$

In the equation, $\partial n_e(t, r, z)$ represents the density of free electrons, where t is the laser's lasting time, r is the distance to the beam's axis, z is depth from the surface to the bulk material.

²Chichkov B N, Momma C, Nolte S, et al., Femtosecond, picosecond and nanosecond laser ablation of solid[J]. Applied Physics A, 1996, 63(2): 109-115

³Du D, Liu X, Korn G, et al. Laser-induced breakdown by impact ionization in SiO₂ with pulse widths from 7 ns to 150 fs[J]. Applied Physics Letters, 1994, 64(23):3071-3073

$P(I)$, inside the equation, represents the free electrons from class 1, it looks like this, in which the σ_N is the absorption's sectional area:

$$P(I) = \sigma_N \times I(t, r, z)$$

Another equation showed up in the general formula is $\beta(I)$, which is the free electrons from class 2, which can be written as:

$$\beta(I) = \alpha_i \times I(t, r, z)$$

Both equations above have $I(t, r, z)$, which represents the intensity of laser. Therefore, in the next step, we need to take a look at how to get the intensity of Laser.

2.3 Intensity of laser

The intensity of laser was influenced by 4 factors:

2.3.1 the change of total number of particles lead to the change of intensity

$$I(t) = I_0 \times \exp(-4 \ln(\frac{2 \times t^2}{t_p^2}))$$

In this equation, t represents the lasting time, I_0 represents the peak of laser's intensity, and t_p represents the length of laser.

2.3.2 Laser is one kind of Gaussian beam

$$E(r) = E_0 \times \exp(-(\frac{r}{r_0})^2)$$

In this equation, $E(r)$ is the vector of beam, in this case represents the intensity of impulse and r_0 is laser's radius

2.3.3 intensity will be influenced by material's reflectivity

$$I(t, r) = I_0(1 - R(t, r)) \times \exp(-(\frac{r}{r_0})^2 - 4 \times \ln(2) \times \frac{t^2}{t_p^2})$$

In this case, t_p is the length of impulse

2.3.4 intensity will be influenced by material absorption

$$\frac{\partial I(t, r, z)}{\partial z} = -\alpha(t, r, z) \times I(t, r, z)$$

Combining all of those 4 factors above, we can get the final formula:

$$I(t, r, z) = I_0(1 - R(t, r)) \times \exp[-(\frac{r}{r_0})^2] - 4 \ln(2) \frac{t^2}{t_p^2} - \int_0^z \alpha(t, r, z) dz$$

In this equation, I_0 represents the peak of laser's intensity. It can be written as:

$$I_0 = \frac{2F}{\sqrt{\frac{\pi}{\ln 2 \times t_p}}}$$

F is the symbol of laser energy's unit, which can be written as:

$$F = \frac{2E}{\pi \times (r_0)^2}$$

If we take a look at this equation, E represents the energy of beam, r_0 represents the radius, which is $r_0 = \frac{W}{2} = \frac{4 \times L \times \lambda}{\pi \times D}$
(L is beam's waist, λ is center's wavelength, and D is the diameter)

2.4 Reflectivity

Now we have the basic formula of the intensity of laser. However, inside this equation, we still need to know the formula of reflectivity and material's absorption. Let's take a look at reflectivity first.

reflectivity(R(t, r)) consists of 2 parts as well:

- steady-state reflectivity: when laser reaches the surface
- Plasma reflectivity: after the surface was ionized

For simplicity, we use the reflectivity formula from Fresnel expression directly in this project:

$$R(r, t) = \frac{(n - 1)^2 + k^2}{(n + 1)^2 + k^2}$$

In this function, n and k are the real part and the imaginary part of reflectivity, which can be written as:

$$n = \sqrt{\frac{\epsilon_r + \sqrt{\epsilon_r^2 + \epsilon_i^2}}{2}}$$

$$k = \sqrt{\frac{-\epsilon_r + \sqrt{\epsilon_r^2 + \epsilon_i^2}}{2}}$$

ϵ_r & ϵ_i are real part and imaginary part of plasma dielectric function. They can be expanded as:

$$\epsilon_r = \left(1 - \frac{\omega_p^2(n_e) \times r^2}{1 + \omega^2 \times \theta^2}\right)$$

$$\epsilon_i = \frac{\omega_p^2(n_e) \times \theta}{\omega \times (1 + \omega^2 \times \theta^2)}$$

Inside these 2 functions, ω represents the laser's frequency; θ represents the free electron's relaxation time, here we assume it's 100fs; $\omega = \frac{2 \times \pi \times c}{\lambda}$ (c is the speed

of light, and λ is the wavelength of laser);
 ω_p represents the plasma's frequency, which can be written as

$$\omega_p(n_e) = \sqrt{\frac{n_e(t, r, z) \times e^2}{m_e \times \epsilon_0}}$$

2.5 Absorption

Firstly, we need to calculate the material's thermal absorption:

$$\alpha_h = \frac{4\pi \times k}{\lambda}$$

Then we use α_h to calculate total absorption:

$$\alpha(t, r, z) = \alpha_h + \alpha_i \times n_e(t, r, z) \times U_1$$

3 Program

Now we have a complete formula to calculate the density of free electrons. We use this formula to write our simulate program. Here's how program does:

- At the beginning, femtosecond laser has not reached the material \rightarrow no free electrons $\rightarrow n_{e1}(t, r, z) = 0$
- When the femtosecond just reached the surface, $n_{e1}(t, r, z) = 0 \rightarrow$ we can calculate $\omega_{p1}(n_e)$
- Based on the formula, we can calculate ϵ_{i1} and ϵ_{r1}
- Based on these 2 values, we can calculate both real part and imaginary part of reflectivity k_1 and n_1
- After that, we can calculate initial reflectivity $R_1(t, r)$
- After the laser get into the material, we can use ϵ_{i1} , ϵ_{r1} , k_1 , n_1 to calculate thermal absorption α_{h1}
- Base on the α_{h1} , we can calculate $\alpha_1(t, r, z)$
- With the increasing of time, we can assume $\alpha_1(t, r, z)$ and $R_1(t, r)$ stay the same but density of free electron changed
- We can calculate $n_{e2}(t, r, z)$ based on the $\alpha_1(t, r, z)$ and $R_1(t, r)$
- repeat these steps to get $R_2(t, r)$ & $\alpha_1(t, r, z) \rightarrow n_{e3}(t, r, z) \rightarrow \dots$

4 Graph

After simplification, we discover that the formulas of β and Π are similar in structure, with only one parameter different. We put aside the parameter and made a 3D image of the two functions changing through the laser impulse time. We can see that the image is decreasing on the Z axis, but the value of the entire plane is always greater than 0. This means that (partial derivative n_e /partial derivative t) increases through time, while its second derivative shows a decreasing trend. That is to say, as the laser irradiation time increases, the free electron density also increases, but the increase rate gradually slows down.

In the program part, we wrote two main ODE functions, one is used to find the default n_e matrix, and the other is used to find the result of the n_e matrix. In order to solve it more efficiently, we use Euler's method to find the numerical solution of the equation instead of the analytical one, and calculate the average free electron density in the small chunks with an accuracy of one μm grid by grid. Finally, the expected matrix is generated.

Reference

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- <http://schen.ucsd.edu/lab/papers/paper45.pdf> – reflectivity formula